

DHLLDV Framework An Overview Of Slurry Transport Models

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Dredging A Way Of Life



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SLURRY TRANSPORT



Fundamentals, A Historical Overview & The Delft Head Loss & Limit Deposit Velocity Framework



Goals & Targets

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Pump/Pipeline System



- Total pressure/power required
- Limit (Stationary) Deposit Velocity
- Cavitation limit of each pump

lacksquare

Delft University of Technology Offshore & Dredging Engineering Deposition/plugging the pipeline

Pressure/Flow Graph (Q-H Graph)



Working points/working area in a stationary situation

0

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Goals & Targets

Determining Slurry Transport Behavior Based On Known Parameters Like: **Liquid Properties, Pipe Diameter, Particle Diameter, Volumetric Concentration As A Function Of The Flow Or** Line Speed

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Possibilities

- **1. Small versus large pipe diameter**
- 2. Small versus large particle diameter
- **3. Low versus high concentration**
- 4. Low versus high line speed
- 5. Spatial versus delivered concentration
- 6. Uniform versus graded sands/gravels
- **1. Carrier liquid properties**
- 2. Solids properties

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For sands/gravels in water 64 combinations possible



The Solids Effect

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Solids Effect



Data from Yagi et al., i_m-v_{ls}





Data looks unorganized depending on the volumetric concentration of the solids.

Data from Yagi et al., E_{rhg}-i_l



FUDDEIFT Delft University of Technology Offshore & Dredging Engineering Data looks more organized not depending on the volumetric concentration of the solids.

Spatial versus Transport Concentration & the Slip Velocity

Spatial Volumetric Concentration is volume based. Transport Volumetric Concentration is volume flow based.

$$\mathbf{C}_{\mathrm{vt}} = \left(1 - \frac{\mathbf{v}_{\mathrm{sl}}}{\mathbf{v}_{\mathrm{ls}}}\right) \cdot \mathbf{C}_{\mathrm{vs}} \quad \Rightarrow \quad \mathbf{C}_{\mathrm{vt}} < \mathbf{C}_{\mathrm{vs}} \quad \mathbf{C}_{\mathrm{vs}} = \left(\frac{\mathbf{v}_{\mathrm{ls}}}{\mathbf{v}_{\mathrm{ls}} - \mathbf{v}_{\mathrm{sl}}}\right) \cdot \mathbf{C}_{\mathrm{vt}}$$

Relative Excess Hydraulic Gradient E_{rhg} , C_{vt} =constant.

$$\mathbf{E}_{\mathrm{rhg}} = \frac{\mathbf{i}_{\mathrm{m}} - \mathbf{i}_{\mathrm{l}}}{\mathbf{R}_{\mathrm{sd}} \cdot \left(1 - \frac{\mathbf{v}_{\mathrm{sl}}}{\mathbf{v}_{\mathrm{ls}}}\right) \cdot \mathbf{C}_{\mathrm{vs}}} = \left(\frac{\mathbf{v}_{\mathrm{ls}}}{\mathbf{v}_{\mathrm{ls}} - \mathbf{v}_{\mathrm{sl}}}\right) \cdot \frac{\mathbf{i}_{\mathrm{m}} - \mathbf{i}_{\mathrm{l}}}{\mathbf{R}_{\mathrm{sd}} \cdot \mathbf{C}_{\mathrm{vs}}}$$



The slip velocity here is the velocity difference between the line speed and the particle velocity.









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Regimes History



The 5 Main Flow Regimes

The 5 main flow regimes are identified based on their dominating behavior regarding energy dissipation.

- 1. The fixed bed regime is identified based on shear stresses at the liquid-fixed bed interface (sheet flow).
- 2. The sliding bed regime is identified based on sliding friction energy losses.
- 3. The heterogeneous flow regime is identified based on potential and kinetic energy losses.
- 4. The homogeneous flow regime is identified based on energy losses in turbulent eddies and viscous friction.
- 5. The sliding flow regime is identified based on sliding friction, potential and kinetic energy losses.At flow regime transitions, a mix of two flow regimes will be present.

The Solids Effect Graph

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How To Read The Graph? (D_p=6 inch)



Different Models Fine Sand

Relative excess hydraulic gradient E_{rhg} vs. Hydraulic gradient i_l

👄 👄 Slidina Bed Cvs=c. Mean 1.000 elative excess hydraulic gradient E_{rhg} (-) Equivalent Liquid Model Homogeneous Flow 0.100 Cvs=Cvt=c. Resulting Erhg curve Cvs=c. 0.010 Resulting Erha curve Cvt=c. X Limit Deposit Velocity 0.001 0.010 0.100 1.000 0.001 Hydraulic gradient i₁ (-) Dp=0.7620 m, d=0.200 mm, Rsd=1.585, Cv=0.300, µsf=0.416

4 possible models: Black heterogeneous, blue pseudo homogeneous, light brown pseudo homogeneous & red homogeneous.

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D-HL-LDV



Different Models Coarse Sand & Gravel

Relative excess hydraulic gradient E_{rhg} vs. Hydraulic gradient i_l



5 possible models: Orange SB/He, black He, blue pseudo Ho, light brown pseudo Ho & red Ho.

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D-HL-LDV





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Zandi & Govatos, Yagi et al. & Babcock





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22 Models i_m-v_{ls} graph



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For small pipe diameters the models are still "close". For large diameter pipes the difference is much much more. **Delft University of Technology – Offshore & Dredging Engineering**

22 Models E_{rhg}-i_l graph





This graph organizes the models better, but there is still a lot of difference between the models.

Types of Models

- There are many empirical models, mainly for heterogeneous flow, some for sliding bed and homogeneous flow.
- Most empirical models add one term to the Darcy-Weisbach equation, often based on Froude numbers.
- There is the equivalent liquid model (ELM) for homogeneous flow.
- There are some 2 layer and 3 layer models for transport with a stationary or sliding bed or sheet flow, Wilson, Doron & Barnea, SRC Model, Matousek.
- The 2 layer and 3 layer models are closed with
 empirical equations for the bed shear stress and the
 concentration distribution.

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Stationary/Fixed Bed Regime Chapter 7.3 & 8.3

Wilson et al. Doron & Barnea SRC Model Matousek Model DHLLDV Framework

Definitions



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ft

ρ

Equilibrium of Forces



$$\Delta p = \Delta p_2 = \Delta p_1 = \frac{\tau_{1,l} \cdot O_1 \cdot \Delta L + \tau_{12,l} \cdot O_{12} \cdot \Delta L}{A_1} = \frac{F_{1,l} + F_{12,l}}{A_1}$$

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Kazanskij (1980), Cvs=0.17



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Wilson et al. Doron & Barnea SRC Model Matousek Model DHLLDV Framework

Definitions



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Equilibrium of Forces



- For the wall friction the standard Darcy Weisbach friction coefficient is used (Moody diagram).
 - The Newitt et al. Model is empirical and based on experimental data. Newitt et al. were the first to use the solids effect graph.
 - The Wilson 2LM is based on a bed and water above. The bed friction is the Darcy Weisbach friction coefficient with the particle diameter as the roughness multiplied with a factor. Televantos found a factor 2, but Wilson also used different factors over the years like 2.6. For the normal stress between the bed and the wall Wilson uses a hydrostatic stress distribution, resulting in a higher friction force compared to the submerged weight times the sliding friction coefficient.

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- The Wilson model is based on constant spatial concentration. Constant delivered concentration curves are constructed by interpolation.
- Doron & Barnea (2LM) basically use the Wilson model, but extended it with suspension above the bed, based on the standard advection diffusion equation. For the constant delivered concentration case this always results in a sliding bed, also at very low line speeds. So they extended their model to a 3LM giving it the possibility to have a fixed bed at very low line speeds.
 - The SRC model is based on the Wilson model for constant spatial concentration, but with suspension above the bed. The fraction in suspension and the fraction in the bed are based on an exponential power.

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- This power contains the ratio of the terminal settling velocity to the line speed. The suspended fraction forms an adjusted carrier liquid, resulting in adjusted liquid properties and an adjusted submerged weight of the bed. This way there is a smooth transition of fully stratified flow to heterogeneous flow to homogeneous flow.
- Matousek uses a completely different method. Based on the delivered concentration, the Shields parameter is determined with the reversed Meyer Peter Muller equation. Once the Shields parameter is known, the bed friction coefficient can be determined from the equivalent bed roughness. The method is based on sheet flow as a transport mechanism.

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The DHLLDV Framework uses the Wilson approach, however with the weight approach for the sliding friction. So the sliding friction force equals the submerged weight times the sliding friction coefficient. Above the bed sheet flow is assumed according to Wilson & Pugh. The method is spatial concentration based. The delivered concentration follows from the transport in the sheet flow layer and the transport in the sliding bed. The method results in a solids effect almost equal to the sliding friction coefficient.

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The Submerged Weight Approach



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π

The Limit of Stationary Deposit Velocity



$$\mathbf{F}_{2,\mathrm{sf}} + \mathbf{F}_{2,\mathrm{l}} = \mathbf{F}_{12,\mathrm{sf}} + \Delta \mathbf{p} \cdot \mathbf{A}_2$$



The E_{rhg} Value is almost μ_{sf}



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Resulting Hydraulic Gradient Graph, C_{vt}



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Wiedenroth (1967), Medium Sand



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Wiedenroth (1967), Coarse Sand



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Heterogeneous Flow Regime Chapter 7.5 & 8.5

Durand & Condolios Newitt et al. Jufin & Lopatin Fuhrboter – Wilson et al. DHLLDV Framework

Existing Equations Depending on i

$$\Delta p_{m} = \Delta p_{l} \cdot \left(1 + \Phi \cdot C_{vt}\right) \quad \text{with:} \quad \Phi = \frac{i_{m} - i_{l}}{i_{l} \cdot C_{vt}} = \frac{\Delta p_{m} - \Delta p_{l}}{\Delta p_{l} \cdot C_{vt}}$$

Durand, Condolios & Gibert based on Froude numbers

$$\Phi = \mathbf{K} \cdot \psi^{-3/2} = \mathbf{K} \cdot \left(\frac{\mathbf{v}_{ls}^2}{\mathbf{g} \cdot \mathbf{D}_p \cdot \mathbf{R}_{sd}} \cdot \sqrt{\mathbf{C}_x} \right)^{-3/2} \quad \text{with:} \quad \mathbf{K} \approx 85$$

Newitt et al. based on potential energy losses

$$\Delta \mathbf{p}_{m} = \Delta \mathbf{p}_{l} \cdot \left(1 + \mathbf{K}_{1} \cdot \left(\mathbf{g} \cdot \mathbf{D}_{p} \cdot \mathbf{R}_{sd} \right) \cdot \mathbf{v}_{t} \cdot \mathbf{C}_{vt} \cdot \left(\frac{1}{\mathbf{v}_{ls}} \right)^{3} \right) \qquad \mathbf{K}_{1} = 1100$$

Jufin & Lopatin empirical large diameters

$$\Delta p_{m} = \Delta p_{l} \cdot \left(1 + 2 \cdot \left(\frac{v_{min}}{v_{ls}} \right)^{3} \right) \implies v_{min} = 5.5 \cdot \left(C_{vt} \cdot \psi^{*} \cdot D_{p} \right)^{1/6}$$

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Existing Equations Independent of i

Fuhrboter medium diameters

$$\Delta p_{m} = \Delta p_{l} + \rho_{l} \cdot g \cdot \Delta L \cdot \frac{S_{k}}{v_{ls}} \cdot C_{vs}$$
$$i_{m} - i_{l} = \frac{S_{k}}{v_{ls}} \cdot C_{vs} \implies E_{rhg} = \frac{i_{m} - i_{l}}{R_{sd} \cdot C_{vs}} = \frac{S_{k}}{R_{sd} \cdot v_{ls}}$$

Wilson heterogeneous empirical (Stratification Ratio)

$$\Delta \mathbf{p}_{m} = \Delta \mathbf{p}_{l} + \frac{\mu_{sf}}{2} \cdot \rho_{l} \cdot \mathbf{g} \cdot \mathbf{R}_{sd} \cdot \Delta \mathbf{L} \cdot \left(\frac{\mathbf{v}_{50}}{\mathbf{v}_{ls}}\right)^{M} \cdot \mathbf{C}_{vt}$$

$$\mathbf{i}_{m} - \mathbf{i}_{l} = \frac{\mu_{sf}}{2} \cdot \mathbf{R}_{sd} \cdot \left(\frac{\mathbf{v}_{50}}{\mathbf{v}_{ls}}\right)^{M} \cdot \mathbf{C}_{vt} \implies \mathbf{E}_{rhg} = \frac{\mu_{sf}}{2} \cdot \left(\frac{\mathbf{v}_{50}}{\mathbf{v}_{ls}}\right)^{M} = \mathbf{R}$$

DHLLDV Framework

Energy Dissipation by:

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- Turbulence Viscous Dissipation (Darcy Weisbach)
- Potential Energy Losses (Hindered Settling Velocity)
- Kinetic Energy Losses (Collisions)

$$\Delta p_{m} = \Delta p_{l,visc} + \Delta p_{s,kin} = \Delta p_{l,visc} \cdot \left[1 + \frac{\Delta p_{s,pot}}{\Delta p_{l,visc}} + \frac{\Delta p_{s,kin}}{\Delta p_{l,visc}}\right]$$

$$\frac{\Delta p_{m}}{\Delta L} = \lambda_{l} \cdot \frac{1}{D_{p}} \cdot \frac{1}{2} \cdot \rho_{l} \cdot v_{ls}^{2} + \rho_{l} \cdot g \cdot R_{sd} \cdot C_{vs} \cdot \left(\frac{v_{t} \cdot (1 - C_{vs} / \kappa_{C})^{\beta}}{v_{ls}}\right) + \rho_{l} \cdot g \cdot R_{sd} \cdot C_{vs} \cdot \left(\frac{v_{sl}}{v_{t}}\right)^{2}$$

$$\Delta \mathbf{p}_{m} = \Delta \mathbf{p}_{l} \cdot \left(1 + \frac{\left(2 \cdot \mathbf{g} \cdot \mathbf{R}_{sd} \cdot \mathbf{D}_{p} \right)}{\lambda_{l}} \cdot \mathbf{C}_{vs} \cdot \frac{1}{v_{ls}^{2}} \cdot \left(\frac{\mathbf{v}_{t} \cdot \left(1 - \mathbf{C}_{vs} / \kappa_{C} \right)^{\beta}}{v_{ls}} + \left(\frac{\mathbf{v}_{sl}}{\mathbf{v}_{t}} \right)^{2} \right) \right)$$
Slip

Verification & Validation, Durand et al.



Durand & Condolios (1952)

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Verification & Validation, Clift et al.



Clift (1982)







Homogeneous Flow Regime Chapter 7.6 & 8.6

Equivalent Liquid Model Newitt et al. Wilson et al. Talmon DHLLDV Framework





The Equivalent Liquid Model (ELM)

$$\begin{split} \Delta \mathbf{p}_{m} &= \lambda_{1} \cdot \frac{\Delta L}{D_{p}} \cdot \frac{1}{2} \cdot \mathbf{p}_{m} \cdot \mathbf{v}_{ls}^{2} \\ \mathbf{i}_{m} &= \frac{\Delta \mathbf{p}_{m}}{\mathbf{p}_{l} \cdot \mathbf{g} \cdot \Delta L} = \frac{\mathbf{p}_{m}}{\mathbf{p}_{l}} \cdot \frac{\lambda_{l} \cdot \mathbf{v}_{ls}^{2}}{2 \cdot \mathbf{g} \cdot \mathbf{D}_{p}} \\ &= \mathbf{i}_{l} \cdot \left(1 + \mathbf{A} \cdot \mathbf{R}_{sd} \cdot \mathbf{C}_{vs}\right) \\ \mathbf{E}_{rhg} &= \frac{\mathbf{i}_{m} - \mathbf{i}_{l}}{\mathbf{R}_{sd} \cdot \mathbf{C}_{vs}} = \mathbf{A} \cdot \mathbf{i}_{l} \end{split}$$

Phenomena

- Very fine particles: The liquid properties have to be adjusted. The ELM can be used with the adjusted liquid properties.
- Fine particles: The ELM can be used with the original liquid properties. At high line speeds the lubrication effect will be mobilized.
- Medium/Coarse particles: The lubrication effect is mobilized, due to a particle poor viscous sub-layer. This gives a reduction of the solids effect in the ELM.



X=1

$$\frac{\overline{\operatorname{Stk} \cdot 9 \cdot \rho_{1} \cdot \nu_{1} \cdot D_{p}}}{\rho_{s} \cdot v_{\mathrm{ls,ldv}}} \approx \sqrt{\frac{\operatorname{Stk} \cdot 9 \cdot \rho_{1} \cdot \nu_{1} \cdot D_{p}}{\rho_{s} \cdot 7.5 \cdot D_{p}^{0.4}}}$$

$$\rho_{x} = \rho_{1} + \rho_{1} \cdot \frac{X \cdot C_{vs} \cdot K_{sd}}{\left(1 - C_{vs} + C_{vs} \cdot X\right)}$$

VC

 $d_{\text{lim}} = 1$

$$C_{vs,x} = \frac{X \cdot C_{vs}}{\left(1 - C_{vs} + C_{vs} \cdot X\right)} \quad \text{and} \quad C_{vs,r} = \left(1 - X\right) \cdot C_{vs}$$

$$u_{x} = \mu_{1} \cdot \left(1 + 2.5 \cdot C_{vs,x} + 10.05 \cdot C_{vs,x}^{2} + 0.00273 \cdot e^{16.6 \cdot C_{vs,x}} \right)$$

$$v_x = \frac{\mu_x}{\rho_x}$$
 and $R_{sd,x} = \frac{\rho_s - \rho_x}{\rho_x}$



- Newitt et al. use a factor A=0.6 in the ELM.
- Wilson et al. Use different factors for A in the ELM.
- Talmon determined A based on a particle free viscous sublayer in 2D channel flow.
- The DHLLDV Framework determined A based on a concentration distribution in a circular pipe. This way the viscous sublayer is particle poor, but not completely particle free. The result is an equation for A, depending on the concentration.
- The DHLLDV Framework also assumes that particles fitting in the viscous sublayer do not result in a particle free viscous sublayer and thus have A=1. The larger the particles the more the particle free sublayer is mobilised.

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Fine Particles

$$\mathbf{E}_{\mathbf{rhg}} = \frac{\mathbf{i}_{\mathbf{m}} - \mathbf{i}_{\mathbf{l}}}{\mathbf{R}_{\mathbf{sd}} \cdot \mathbf{C}_{\mathbf{vs}}} = \mathbf{i}_{\mathbf{l}} \cdot \left(1 - \left(1 + \mathbf{R}_{\mathbf{sd}} \cdot \mathbf{C}_{\mathbf{vs}} - \left(\frac{\mathbf{A}_{\mathbf{C}_{\mathbf{v}}}}{\kappa} \cdot \ln\left(\frac{\mathbf{\rho}_{\mathbf{m}}}{\mathbf{\rho}_{\mathbf{l}}}\right) \cdot \sqrt{\frac{\lambda_{\mathbf{l}}}{8}} + 1 \right)^2}{\mathbf{R}_{\mathbf{sd}} \cdot \mathbf{C}_{\mathbf{vs}} \cdot \left(\frac{\mathbf{A}_{\mathbf{C}_{\mathbf{v}}}}{\kappa} \cdot \ln\left(\frac{\mathbf{\rho}_{\mathbf{m}}}{\mathbf{\rho}_{\mathbf{l}}}\right) \cdot \sqrt{\frac{\lambda_{\mathbf{l}}}{8}} + 1 \right)^2} \right) \cdot \left(1 - \left(\frac{\delta_{\mathbf{v}}}{\mathbf{d}} \right) \right) \right)$$

$$\mathbf{E}_{\mathrm{rhg}} = \frac{\mathbf{i}_{\mathrm{m}} - \mathbf{i}_{\mathrm{l}}}{\mathbf{R}_{\mathrm{sd}} \cdot \mathbf{C}_{\mathrm{vs}}} = \mathbf{i}_{\mathrm{l}} \cdot \left(1 - \left(1 - \alpha_{\mathrm{E}}\right) \cdot \left(1 - \left(\frac{\delta_{\mathrm{v}}}{\mathrm{d}}\right)\right) \right)$$

$$\mathbf{m} = \mathbf{i}_{l} + \mathbf{i}_{l} \cdot \mathbf{R}_{sd} \cdot \mathbf{C}_{vs} \cdot \left(1 - \left(1 - \alpha_{E}\right) \cdot \left(1 - \left(\frac{\delta_{v}}{d}\right)\right)\right)$$

$$\left(\frac{\delta_{v}}{d}\right) = \max = 1 \quad \Rightarrow \quad i_{m} = i_{l} + i_{l} \cdot R_{sd} \cdot C_{vs} = i_{l} \cdot \left(1 + R_{sd} \cdot C_{vs}\right) \quad \text{ELM}$$

$$\left(\frac{\delta_{v}}{d}\right) = 0 \quad \Rightarrow \quad i_{m} = i_{l} + i_{l} \cdot R_{sd} \cdot C_{vs} \cdot \alpha_{E} = i_{l} \cdot \left(1 + R_{sd} \cdot C_{vs} \cdot \alpha_{E}\right)$$

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Medium/Coarse Particles

$$\begin{split} \mathbf{E}_{\mathbf{rhg}} &= \mathbf{i}_{l} \cdot \frac{1 + \mathbf{R}_{\mathbf{sd}} \cdot \mathbf{C}_{\mathbf{vs}} - \left(\frac{\mathbf{A}_{\mathbf{C}_{\mathbf{v}}}}{\kappa} \cdot \ln\left(\frac{\mathbf{\rho}_{\mathbf{m}}}{\mathbf{\rho}_{l}}\right) \cdot \sqrt{\frac{\lambda_{l}}{8}} + 1\right)^{2}}{\mathbf{R}_{\mathbf{sd}} \cdot \mathbf{C}_{\mathbf{vs}} \cdot \left(\frac{\mathbf{A}_{\mathbf{C}_{\mathbf{v}}}}{\kappa} \cdot \ln\left(\frac{\mathbf{\rho}_{\mathbf{m}}}{\mathbf{\rho}_{l}}\right) \cdot \sqrt{\frac{\lambda_{1}}{8}} + 1\right)^{2}} = \alpha_{\mathbf{E}} \cdot \mathbf{i}_{l} \\ \mathbf{i}_{\mathbf{m}} &= \mathbf{i}_{l} + \mathbf{i}_{l} \cdot \frac{1 + \mathbf{R}_{\mathbf{sd}} \cdot \mathbf{C}_{\mathbf{vs}} - \left(\frac{\mathbf{A}_{\mathbf{C}_{\mathbf{v}}}}{\kappa} \cdot \ln\left(\frac{\mathbf{\rho}_{\mathbf{m}}}{\mathbf{\rho}_{l}}\right) \cdot \sqrt{\frac{\lambda_{1}}{8}} + 1\right)^{2}}{\left(\frac{\mathbf{A}_{\mathbf{C}_{\mathbf{v}}}}{\kappa} \cdot \ln\left(\frac{\mathbf{\rho}_{\mathbf{m}}}{\mathbf{\rho}_{l}}\right) \cdot \sqrt{\frac{\lambda_{1}}{8}} + 1\right)^{2}} \\ \mathbf{p}_{\mathbf{m}} &= \mathbf{p}_{l} + \mathbf{p}_{l} \cdot \frac{1 + \mathbf{R}_{\mathbf{sd}} \cdot \mathbf{C}_{\mathbf{vs}} - \left(\frac{\mathbf{A}_{\mathbf{C}_{\mathbf{v}}}}{\kappa} \cdot \ln\left(\frac{\mathbf{\rho}_{\mathbf{m}}}{\mathbf{\rho}_{l}}\right) \cdot \sqrt{\frac{\lambda_{1}}{8}} + 1\right)^{2}}{\left(\frac{\mathbf{A}_{\mathbf{C}_{\mathbf{v}}}}{\kappa} \cdot \ln\left(\frac{\mathbf{\rho}_{\mathbf{m}}}{\mathbf{\rho}_{l}}\right) \cdot \sqrt{\frac{\lambda_{1}}{8}} + 1\right)^{2}} \end{split}$$

Lubrication Factor α_E



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Very Fine Particles



Thomas (1976)



Very Fine Particles, with Thomas (1965)





Thomas (1976) Adjusted Liquid Properties

Fine Particles



Whitlock (2004)







Blythe & Czarnotta (1995)





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SRC Model DHLLDV Framework

Phenomena

If the particle diameter to pipe diameter is larger than about 0.015, the particles will not be suspended anymore, but stay in a fast flowing sort of bed.

The behavior is a mix of the heterogeneous flow regime and the sliding bed regime.

At $d/D_p=0.015$ the behavior is still heterogeneous, but the larger the particle diameter the more it is sliding bed behavior.

The higher the line speed the smaller the concentration of the flowing particles at the bottom of the pipe.

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Verification & Validation, Boothroyde



Boothroyde et al. (1979)

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Verification & Validation, Wiedenroth



Wiedenroth (1967)







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Conclusions

Models are valid for the parameters the experiments were carried out with.

The correct flow regime has to be identified for each (sub) model.

For heterogeneous flow the solids effect is independent of the Darcy Weisbach component.

Models validated with a wide range of parameters are: Jufin & Lopatin, Wilson et al., the SRC model and the DHLLDV Framework.

These 4 models give similar results for medium and coarse sands over a wide range of pipe diameters.

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Fundamentals, A Historical Overview & The Delft Head Loss & Limit Deposit Velocity Framework



The Elephant of Wilson is our best Friend



