A MODEL CALCULATION FOR FLOW RESISTANCE IN THE HYDRAULIC TRANSPORT OF SAND

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ABSTRACT

The hydraulic transport of sand is an important process in dredging engineering. An accurate calculation of the flow resistance of sand when hydraulically transported is a significant part of any reasonable design for a pump-pipeline transport system. Although there have been a number of empirical formulae proposed by Durand, Condolios, Jufin, Wilson, GIW, etc., the flow resistance calculation still needs to be subjected to further investigation. A model that is widely used in coal hydraulic transport, called the Wasp model is introduced in this paper. The model classifies particles as homogeneously distributed particles and heterogeneously distributed bed particles. Water and the suspended particles form the so called “two-phase carrying fluid” or “vehicle” and transport the heterogeneous bed particles. This paper tries to apply a model, proposed by Wasp and used in coal slurry, to the resistance calculation for the hydraulic transport of sand. The model is checked by our measured data from the laboratory stand of sand transport at Delft University of Technology, The Netherlands. Results show that the model can result in a reasonable prediction for the resistance of small particles slurry. However, like the other popular models, it could not give a satisfactory prediction for coarse sand transport either. The accuracy given by the Wasp model actually depends on the modeling of the homogeneous slurry and the modeling of the heterogeneous slurry.

Keywords: Sand hydraulic transport, flow resistance, Wasp model.

INTRODUCTION

Solids transported in dredging engineering can be mud, clay, sand, gravel, coral reef or other materials. Different mechanisms are active in the solid-liquid mixture flow, which has led in a great number of empirical models to describe the mechanisms governing the slurry flow regime in the past decades (Durand 1953a,b, Wilson 1976, 1997, Matousek 1997). Generally speaking, solids hydraulic transport can be conceptually divided into fully-stratified, partially-stratified and fully-suspended flow (Sundqvist et al. 1997). The most common type of slurry flow is the partially-stratified or heterogeneous case, in which turbulent suspension and inter-granular contact are the two significant mechanisms of particle support. Due to the complexity of slurry flow, however, the modeling of the interaction between solids and the carrying liquid is still subject to further study. That is why the modeling of slurry flow resistance has always been an important subject in dredging and other solid hydraulic transport industries. This paper tries to apply a model, proposed by Wasp and used in coal slurry, to the resistance calculation for the hydraulic transport of sand. The model is checked by our measured data from the laboratory stand of sand transport at Delft

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Division of Slurry Flow

Based on coal slurry data accumulated over 13 years of experiments and actual 102-mile pipeline transport, Wasp proposed to separate the slurry flow into two components: a vertically homogeneous slurry flow called a “vehicle” and a vertically heterogeneous slurry called a “Durand” flow (Wasp 1963, 1977, Onishi et al. 2002). Water and the suspended smaller particles form the so called “two-phase carrying fluid” or “vehicle” and transports the heterogeneous coarser particles. According to the model, the total pressure loss per unit pipe length $J$ is the sum of the losses due to the “vehicle” and the “Durand” flow components:

$$J = J_{du} + J_{ve}$$

where $J_{du}$ and $J_{ve}$ denote the pressure loss gradients of the “vehicle” and the heterogeneous slurry, respectively.

The most important step in the use of the Wasp model is to determine which portion of the slurry is in the “vehicle” and which is in the “Durand” heterogeneous flow portion. The “vehicle” volume concentration $C_{ve}$ and the “Durand” heterogeneous volume portion concentration $C_{du}$ are divided from the slurry total volume concentration $C_{vt}$ as follows:

$$C_{ve} = C_{vt} \left( \frac{C_{top}}{C_{mid}} \right)$$

where $C_{top}/C_{mid}$ determines the amount of the suspended particles contributed to the “vehicle”. Here $C_{top}$ and $C_{mid}$ denote the solid volume fraction at $y/D=0.92$ and at $y/D=0.5$ of the pipe cross-section, respectively, where $y$ and $D$ denote the vertical position from the pipe bottom and the diameter of the pipe, respectively. The following equation is usually used to determine the division of the two portions:

$$\log \frac{C_{top}}{C_{mid}} = -1.8(\frac{\omega}{\beta ku_*})$$

where $\omega$ denotes solid settling velocity, $\beta$ constant (=1), $\kappa$ the Von Karman constant (=0.4 in this paper), and
V \_s \text{ the friction velocity, given by } V_\varepsilon = V_m \sqrt{\frac{\lambda_{ve}}{8}}, \text{ where } \lambda_{ve} \text{ denotes the friction coefficient of the “vehicle” and } V_m \text{ the mean velocity of the total slurry.}

**Resistance Calculation of the Two Flow Portions**

The resistance caused by the “vehicle” is calculated by

\[
J_{ve} = \frac{\lambda_{ve} V_m^2}{2D} \rho_{ve}
\]

where \( \rho_{ve} \) denotes the density of the “vehicle”. The friction coefficient \( \lambda_{ve} \) can be obtained from the Moody diagram, depending on the flow regime of the “vehicle”. It is usually considered that the “vehicle” flow is in the smooth turbulent regime and \( \lambda_{ve} \) can be expressed by

\[
\lambda_{ve} = 0.0032 + 0.221R_{ve}^{-0.237}
\]

where \( R_{ve} \) denotes the Reynolds number of the vehicle.

The pressure drop due to the “Durand” heterogeneous slurry flow can be calculated by the Durand formula:

\[
J_{du} = 82C_{du}\left[\frac{V_m^2}{gD}\left(\frac{\rho_s - \rho}{\rho}\right)^{-1}\sqrt{C_D}\right]^{1.5} J_{ve}
\]

where \( \rho_s, \rho \) denote the particle and water density, respectively, and \( C_D \) the particle drag coefficient.

**CALCULATION RESULTS AND DISCUSSION**

From 1998 to 1999, a series of sand-water slurry tests were carried out at the laboratory stand of sand transport at Delft University of Technology, The Netherlands. The diameter of the pipeline in the closed pump circuit is 150mm. The tested sands were the fine sand, medium sand and coarse sand with the mass-median diameter of \( d_{50}=0.123\text{mm}, d_{50}=0.372\text{mm}, \text{ and } d_{50}=1.84\text{mm}, \) respectively. The particle size distribution is shown in Figure 1. The test data were employed to check the Wasp model in this paper. The comparison between the calculation results and the measured data are shown in Figures 2, 3 and 4. In those figures \( C_{vd} \) denotes the delivered volumetric concentration of sand. In the model calculation, \( C_{vd} \) takes the value of \( C_{vd} \).
Figure 1. Particle size distribution of the tested sands.

Figure 2. Flow resistance of the fine sand slurry, d_{50}=0.123mm.
Generally speaking, the Wasp model could give reasonable prediction to the slurry of fine and medium sand. However, like the other popular models, it could not give a satisfactory prediction for coarse sand transport either. As for the so-called “double peak” flow resistance occurred in the coarse sand slurry, see Figure 4, the Wasp model
could not describe the mechanism either (Matousek 1997, Ni 2004).

Wasp’s contribution to slurry pipeline transport assessment was to introduce explicitly the concept of two-phase flow to the slurry pipeline transport. But the calculation process in the model is much more complicated than that in other popular models and the accuracy given by the model depends on the following three points:

1. the division of the two portions of “vehicle” and “Durand” slurry;
2. the modeling of the “vehicle” resistance; and
3. the modeling of the “Durand” slurry resistance.

For the fine sand slurry, most of the particles were classified as homogeneous portion and then the density of the vehicle $\rho_{ve}$ increased, which resulted in greater “vehicle” resistance by Formula (6). Figures 2(a) & (b) show that the calculated pressure drop in the fine sand slurry is gradually larger than the measured. For the coarse sand slurry, however, little particles contributed to the “vehicle” from low slurry velocities to high slurry velocities, therefore the “Durand” heterogeneous model actually determines the accuracy of the solid-effect calculation in the Wasp model, see Figures 4(a) & (b).

For the medium sand slurry, a certain amount of particles were considered in the “vehicle”. Table 1 shows that an increasing amount of particles contributes to the “vehicle” with the increase of the mean slurry velocity $V_m$ when delivered volumetric concentration $C_{vd}=25-26\%$, see Figure 3(b). In this paper’s validation calculation, the medium sand slurry is the intermediate case and both the “vehicle” modeling and the “Durand” modeling play important roles. As is known to us, there have been a great number of resistance models for the “vehicle” or homogeneous slurry and the “Durand” or heterogeneous slurry. In order to improve the accuracy of the Wasp model, there would be a great deal of research to do in the division of the two flow portions and the appropriate modeling of flow resistance due to the “vehicle” and the “Durand” heterogeneous slurry with high solids concentration.

**Table 1. The “vehicle” and the “Durand” portions in the medium sand slurry of $C_{vd}=25-26\%$.**

<table>
<thead>
<tr>
<th>Mean slurry velocity $V_m$ (m/s)</th>
<th>3.98</th>
<th>4.99</th>
<th>5.98</th>
<th>6.98</th>
<th>7.97</th>
<th>8.98</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle portion volume concentration $C_{ve}$ (%)</td>
<td>1.6</td>
<td>3.2</td>
<td>5.5</td>
<td>7.6</td>
<td>9.8</td>
<td>10.5</td>
</tr>
<tr>
<td>Durand portion volume concentration $C_{du}$ (%)</td>
<td>23.2</td>
<td>21.7</td>
<td>20.0</td>
<td>18.0</td>
<td>16.1</td>
<td>16.4</td>
</tr>
<tr>
<td>Percentage of the vehicle portion $C_{ve} / C_{vt}$ (%)</td>
<td>6.5</td>
<td>12.9</td>
<td>22.0</td>
<td>29.7</td>
<td>37.8</td>
<td>39.0</td>
</tr>
</tbody>
</table>
REFERENCES


ACKNOWLEDGEMENTS

The authors wish to thank Dr. Matousek and Mr. Zwartbol for the work they carried out on this research. They also thank the Delft University, The Netherlands for giving them the opportunity to carry out this research.