NEW DEVELOPMENTS OF CUTTING THEORIES WITH RESPECT TO DREDGING THE CUTTING OF CLAY AND ROCK

S.A. Miedema¹

ABSTRACT

From literature on this subject it is known that the process of cutting clay or rock is different from that of water saturated sand. Sand is often modeled as a continuum with an internal friction angle and a sand/steel friction angle (external friction angle), but without cohesion and adhesion. Clay is considered to be a continuum with cohesion (internal shear strength) and adhesion (external shear strength), but with an internal friction angle and a clay/steel friction angle (external friction angle) equal to zero. Rock is considered as a continuum with cohesion (internal shear strength), but without external shear strength, so without adhesion. Rock is also considered to have internal and external friction.

Based on the equilibrium of forces on the chip of soil cut, as derived by Miedema 1987 for soil in general, criteria are formulated to predict the failure mechanism when cutting clay or rock. There are 4 failure mechanisms that can be distinguished depending on the type of material to be cut. The shear type for sand, the curling type, flow type and tear type for clay and the flow type and tear type for rock. The flow type is also known as ductile failure and the tear type as brittle failure.

The derived cutting equations, allow the prediction of the failure mechanism and the cutting forces involved. In this paper simplifications have been applied to allow a clear description of the phenomena involved.

THE GENERAL CUTTING PROCESS

Hatamura and Chijiiwa (1975) distinguished three failure mechanisms in soil cutting. The "shear type", the "flow type" and the "tear type". The "shear type" occurs in materials with an angle of internal friction like sand. A fourth failure mechanism can be distinguished, the "curling type", as is known in metal cutting. Although it seems that the curling of the chip cut is part of the flow of the material, whether the "curling type" or the "flow type" occurs depends on several conditions.



Figure 1. The curling type and the flow type.

¹ Associate Professor of Dredging Engineering & Educational Director of Offshore Engineering, Delft University of Technology, Mekelweg 2, 2628CD Delft, The Netherlands, +31-15-2788359, s.a.miedema@tudelft.nl.

Figure 1 illustrates the curling type and the flow type mechanism as they might occur when cutting clay or rock. Figure 2 illustrates the tear type and the shear type mechanism as they might occur when cutting clay or rock (the tear type) or cutting sand (the shear type). To predict which type of failure mechanism will occur under given conditions with specific soil, a formulation for the cutting forces has to be derived. The derivation is made under the assumption that the stresses on the shear plane and the blade are constant and equal to the average stresses acting on the surfaces.



Figure 2. The tear type and the shear type.

The forces acting on a straight blade when cutting soil, can be distinguished as:

- 1. A force normal to the blade N_2 .
- 2. A shear force S_2 as a result of the soil/steel friction N_2 ·tan(δ).
- 3. A shear force A as a result of pure adhesion between the soil and the blade τ_a . This force can be calculated by multiplying the adhesive shear strength τ_a of the soil with the contact area between the soil and the blade.
- 4. A force W_2 as a result of water under pressure on the blade.

These forces are shown in Figure 3. If the forces N_2 and S_2 are combined to a resulting force K_2 and the adhesive force and the water under pressures are known, then the resulting force K_2 is the unknown force on the blade.



Figure 3. The forces on the blade.

Figure 4 illustrates the forces on the layer of soil cut. The forces shown are valid in general. The forces acting on this layer are:

- 1. The forces occurring on the blade as mentioned above.
- 2. A normal force acting on the shear surface N_1 .
- 3. A shear force S_1 as a result of internal fiction $N_1 \cdot tan(\phi)$.
- 4. A force W_1 as a result of water under pressure in the shear zone.
- 5. A shear force C as a result of pure cohesion τ_c . This force can be calculated by multiplying the cohesive shear strength τ_c with the area of the shear plane.
- 6. A gravity force **G** as a result of the weight of the layer cut.
- 7. An inertial force I, resulting from acceleration of the soil.



Figure 4. The forces on the layer cut.

The normal force N_1 and the shear force S_1 can be combined to a resulting grain force K_1 . By taking the horizontal and vertical equilibrium of forces an expression for the force K_2 on the blade can be derived.

The horizontal equilibrium of forces:

$$K_1 \cdot \sin(\beta + \varphi) - W_1 \cdot \sin(\beta) + C \cdot \cos(\beta) + I \cdot \cos(\beta) - A \cdot \cos(\alpha) + W_2 \cdot \sin(\alpha) - K_2 \cdot \sin(\alpha + \delta) = 0$$
(1)

The vertical equilibrium of forces:

$$-K_1 \cdot \cos(\beta + \varphi) + W_1 \cdot \cos(\beta) + C \cdot \sin(\beta) + I \cdot \sin(\beta) + G + A \cdot \sin(\alpha) + W_2 \cdot \cos(\alpha) - K_2 \cdot \cos(\alpha + \delta) = 0$$
(2)

The force K_1 on the shear plane is now:

$$K_{1} = \frac{W_{2} \cdot \sin(\delta) + W_{1} \cdot \sin(\alpha + \beta + \delta) + G \cdot \sin(\alpha + \delta) - I \cdot \cos(\alpha + \beta + \delta) - C \cdot \cos(\alpha + \beta + \delta) + A \cdot \cos(\delta)}{\sin(\alpha + \beta + \delta + \varphi)}$$
(3)

The force \mathbf{K}_2 on the blade is now:

$$K_{2} = \frac{W_{2} \cdot \sin(\alpha + \beta + \varphi) + W_{1} \cdot \sin(\varphi) + G \cdot \sin(\beta + \varphi) + I \cdot \cos(\varphi) + C \cdot \cos(\varphi) - A \cdot \cos(\alpha + \beta + \varphi)}{\sin(\alpha + \beta + \delta + \varphi)}$$
(4)

From equation 4 the forces on the blade can be derived. On the blade a force component in the direction of cutting velocity \mathbf{F}_{h} and a force perpendicular to this direction \mathbf{F}_{v} can be distinguished.

$$F_{h} = -W_{2} \cdot \sin(\alpha) + K_{2} \cdot \sin(\alpha + \delta) + A \cdot \cos(\alpha)$$

$$(5)$$

$$F_{\nu} = -W_2 \cdot \cos(\alpha) + K_2 \cdot \cos(\alpha + \delta) - A \cdot \sin(\alpha)$$
(6)

The normal force on the shear plane is now:

$$N_{1} = \frac{W_{2} \cdot \sin(\delta) + W_{1} \cdot \sin(\alpha + \beta + \delta) + G \cdot \sin(\alpha + \delta) - I \cdot \cos(\alpha + \beta + \delta) - C \cdot \cos(\alpha + \beta + \delta) + A \cdot \cos(\delta)}{\sin(\alpha + \beta + \delta + \varphi)} \cdot \cos(\varphi)$$
(7)

The normal force on the blade is now:

$$N_{2} = \frac{W_{2} \cdot \sin(\alpha + \beta + \varphi) + W_{1} \cdot \sin(\varphi) + G \cdot \sin(\beta + \varphi) + I \cdot \cos(\varphi) + C \cdot \cos(\varphi) - A \cdot \cos(\alpha + \beta + \varphi)}{\sin(\alpha + \beta + \delta + \varphi)} \cdot \cos(\delta)$$
(8)

If the equations (7) and (8) give a positive result, the normal forces are compressive forces. It can be seen from these equations that the normal forces can become negative meaning that a tensile rupture might occur, depending on values for the adhesion and cohesion and the angles involved. The most critical direction where this might occur can be found from the Mohr circle.

WHICH EQUATION AND MECHANISM FOR WHICH KIND OF SOIL?

To avoid confusion between cohesion and adhesion on one side and internal and external friction on the other side, internal and external friction, also named Coulomb friction, depend linearly on normal stresses, internal friction depends on the normal stress between the sand grains and external friction on the normal stress between the sand grains and external friction on the normal stress between the sand grains and external friction on the normal stress between the sand grains and another material, for example steel. In civil engineering internal and external friction are denoted by the angle of internal friction and the angle of external friction, also named the soil/interface friction angle. In mechanical engineering the internal and external friction angles are denoted by the internal and external friction coefficient. If there is no normal stress, there is no shear stress resulting from normal stress, so the friction is zero. Adhesion and cohesion are considered to be the sticky effect between two surfaces. Cohesion is the sticky effect between two surfaces of the same material before any failure has occurred and adhesion is the sticky effect between two different materials, for example adhesive tape. Adhesion and cohesion could be named the external and internal shear strength which are independent from normal stresses. The equations for the resulting shear stresses are:

$$\tau_i = \tau_c + \sigma_{in} \cdot \tan(\varphi) \quad or \quad \tau_i = \tau_c + \sigma_{in} \cdot \mu_i \tag{9}$$

 $\tau_e = \tau_a + \sigma_{en} \cdot \tan(\delta) \quad or \quad \tau_e = \tau_a + \sigma_{en} \cdot \mu_e \tag{10}$

In which:

t _i Internal shear stress	кРа
$\tau_{\rm c}$ Cohesion or internal shear strength	kPa
σ_{in} Internal normal stress	kPa
φ Angle of internal friction	0
μ_i Internal friction coefficient	-
τ_e External shear stress or soil interface shear stress	kPa
τ_a Adhesion or external shear strength	kPa
σ_{en} External normal stress or soil interface normal stress	kPa
δ Angle of external friction or soil/interface friction angle	0
μ_e External friction coefficient	-

Dry Sand

Pure sand is supposed to be cohesion less, meaning it does not have a shear strength or the shear strength is zero and the adhesion is also zero. The shear stresses, internal and external, depend completely on the normal stresses. In dry sand the pores between the sand grains are filled with air and although dilatation will occur due to shearing, Miedema 1987, there will be hardly any generation of pore under pressures because the permeability for air flowing through the pores is high. This means that the cutting forces do not depend on pore pressure forces, nor on adhesion and cohesion, but only on gravity and inertia, resulting in the following set of equations:

The force \mathbf{K}_1 on the shear plane is now:

$$K_{1} = \frac{G \cdot \sin(\alpha + \delta) - I \cdot \cos(\alpha + \beta + \delta)}{\sin(\alpha + \beta + \delta + \varphi)}$$
(11)

The force \mathbf{K}_2 on the blade is now:

$$K_2 = \frac{G \cdot \sin(\beta + \varphi) + I \cdot \cos(\varphi)}{\sin(\alpha + \beta + \delta + \varphi)}$$
(12)

From equation 12 the forces on the blade can be derived. On the blade a force component in the direction of cutting velocity $\mathbf{F}_{\mathbf{h}}$ and a force perpendicular to this direction $\mathbf{F}_{\mathbf{v}}$ can be distinguished.

$$F_{h} = K_{2} \cdot \sin(\alpha + \delta)$$

$$F_{v} = K_{2} \cdot \cos(\alpha + \delta)$$
(13)
(14)

$$K_{2} \cdot \cos(\alpha + \delta) \tag{14}$$

The normal force on the shear plane is now:

$$N_{1} = \frac{G \cdot \sin(\alpha + \delta) - I \cdot \cos(\alpha + \beta + \delta)}{\sin(\alpha + \beta + \delta + \varphi)} \cdot \cos(\varphi)$$
(15)

The normal force on the blade is now:

$$N_2 = \frac{G \cdot \sin(\beta + \varphi) + I \cdot \cos(\varphi)}{\sin(\alpha + \beta + \delta + \varphi)} \cdot \cos(\delta)$$
(16)

Equations 15 and 16 show that the normal force on the shear plane N1 can become negative at very high velocities, which is physically impossible, while the normal force on the blade N2 will always be positive. Under normal conditions the sum of $\alpha+\beta+\delta$ will be greater than 90 degrees in which case the cosine of this sum is negative, resulting in a normal force on the shear plane that is always positive. Only in the case of a small blade angle α , shear angle β and angle of external friction δ , the sum of these angles could be smaller than 90°, but still close to 90° degrees. For example a blade angle of 30° would result in a shear angle of about 30°. A loose sand could have an external friction angle of 20°, so the sum would be 80°. But this is a lower limit for $\alpha+\beta+\delta$. A more realistic example is a blade with an angle of 60°, resulting in a shear angle of about 20° and a medium to hard sand with an external friction angle of 30°, resulting in $\alpha+\beta+\delta=110^\circ$. So for realistic cases the normal force on the shear plane N₁ will always be positive. In dry sand, always the shear type of cutting mechanism will occur.

Water saturated sand

Water saturated sand is also cohesion less, although in literature the phenomenon of water under pressures is sometimes referred to as apparent cohesion. It should be stated however that the water under pressures have nothing to do with cohesion or shear strength. The shear stresses still follow the rules of Coulomb friction. Due to dilatation, a volume increase of the pore volume caused by shear stresses, under pressures develop around the shear plane as described by Miedema 1987, resulting in a strong increase of the grain stresses. Because the permeability of the flow of water through the pores is very low. The stresses and thus the forces are dominated by the phenomenon of dilatancy and gravitation, inertia, adhesion and cohesion can be neglected.

The force K_1 on the shear plane is now:

$$K_1 = \frac{W_2 \cdot \sin(\delta) + W_1 \cdot \sin(\alpha + \beta + \delta)}{\sin(\alpha + \beta + \delta + \varphi)}$$
(17)

The force \mathbf{K}_2 on the blade is now:

$$K_2 = \frac{W_2 \cdot \sin(\alpha + \beta + \varphi) + W_1 \cdot \sin(\varphi)}{\sin(\alpha + \beta + \delta + \varphi)}$$
(18)

From equation 18 the forces on the blade can be derived. On the blade a force component in the direction of cutting velocity $\mathbf{F}_{\mathbf{h}}$ and a force perpendicular to this direction $\mathbf{F}_{\mathbf{v}}$ can be distinguished.

$$F_{h} = -W_{2} \cdot \sin(\alpha) + K_{2} \cdot \sin(\alpha + \delta)$$

$$F_{v} = -W_{2} \cdot \cos(\alpha) + K_{2} \cdot \cos(\alpha + \delta)$$
(19)
(20)

The normal force on the shear plane is now:

$$N_{1} = \frac{W_{2} \cdot \sin(\delta) + W_{1} \cdot \sin(\alpha + \beta + \delta)}{\sin(\alpha + \beta + \delta + \varphi)} \cdot \cos(\varphi)$$
(21)

The normal force on the blade is now:

$$N_2 = \frac{W_2 \cdot \sin(\alpha + \beta + \varphi) + W_1 \cdot \sin(\varphi)}{\sin(\alpha + \beta + \delta + \varphi)} \cdot \cos(\delta)$$
(22)

Equations 21 and 22 show that the normal forces on the shear plane and the blade are always positive. Positive means compressive stresses. In water saturated sand, always the shear type of cutting mechanism will occur.

Clay

Pure clay under undrained conditions follows the $\varphi=0$ concept, meaning that effectively there is no internal friction and thus there is also no external friction. Under drained conditions clay will have some internal friction, although smaller than sand. The reason for this is the very low permeability of the clay. If the clay is compressed with a high strain rate, the water in the pores cannot flow away resulting in the pore water carrying the extra pressure, the grain stresses do not change. If the grain stresses do not change, the shear stresses according to Coulomb friction do not change and effectively there is no relation between the extra normal stresses and the shear stresses, so apparently $\varphi=0$. At very low strain rates the pore water can flow out and the grains have to carry the extra normal stresses, resulting in extra shear stresses. During the cutting of clay, the strain rates, deformation rates, are so big that the internal and external friction angles can be considered to be zero. The adhesive and cohesive forces play a dominant role, so that gravity and inertia can be neglected.

The force K_1 on the shear plane is now:

$$K_1 = \frac{-C \cdot \cos(\alpha + \beta) + A}{\sin(\alpha + \beta)}$$
(23)

The force \mathbf{K}_2 on the blade is now:

$$K_2 = \frac{C - A \cdot \cos(\alpha + \beta)}{\sin(\alpha + \beta)}$$
(24)

From equation 24 the forces on the blade can be derived. On the blade a force component in the direction of cutting velocity $\mathbf{F}_{\mathbf{h}}$ and a force perpendicular to this direction $\mathbf{F}_{\mathbf{v}}$ can be distinguished.

$$F_{h} = K_{2} \cdot \sin(\alpha) + A \cdot \cos(\alpha)$$

$$F_{\nu} = K_{2} \cdot \cos(\alpha) - A \cdot \sin(\alpha)$$
(25)
(26)

The normal force on the shear plane is now:

$$N_1 = \frac{-C \cdot \cos(\alpha + \beta) + A}{\sin(\alpha + \beta)}$$
(27)

The normal force on the blade is now:

$$N_2 = \frac{C - A \cdot \cos(\alpha + \beta)}{\sin(\alpha + \beta)}$$
(28)

Equations 27 and 28 show that both the normal force on the shear plane and the normal force on the blade may become negative. This depends on the ratio between the adhesive and the cohesive forces and on the blade angle α and shear angle β . A negative normal force on the blade will result in the curling type of cutting mechanism, while a negative normal force on the shear plane will result in the tear type of cutting mechanism. If both normal forces are positive, the flow type of cutting mechanism will occur.

Rock

Rock is the collection of materials where the grains are bonded chemically from very stiff clay, sandstone to very hard basalt. It is difficult to give one definition of rock or stone and also the composition of the material can differ strongly. Still it is interesting to see if the model used for sand and clay, which is based on the Coulomb model, can be used for rock as well. Typical parameters for rock are the compressive strength and the tensile strength and

specifically the ratio between those two which is a measure for how fractured the rock is. Rock also has a shear strength and because it consists of bonded grains it will have an internal friction angle and an external friction angle. It can be assumed that the permeability of the rock is very low, so initially the pore pressures do no play a role. This results in a material where gravity, inertia and adhesion can be neglected.

The force K_1 on the shear plane is now:

$$K_1 = \frac{-C \cdot \cos(\alpha + \beta + \delta)}{\sin(\alpha + \beta + \delta + \phi)}$$
(29)

The force \mathbf{K}_2 on the blade is now:

$$K_2 = \frac{C \cdot \cos(\varphi)}{\sin(\alpha + \beta + \delta + \varphi)} \tag{30}$$

From equation 30 the forces on the blade can be derived. On the blade a force component in the direction of cutting velocity $\mathbf{F}_{\mathbf{h}}$ and a force perpendicular to this direction $\mathbf{F}_{\mathbf{v}}$ can be distinguished.

$$F_h = K_2 \cdot \sin(\alpha + \delta) \tag{31}$$

$$F_{\nu} = K_2 \cdot \cos(\alpha + \delta) \tag{32}$$

The normal force on the shear plane is now:

$$N_1 = \frac{-C \cdot \cos(\alpha + \beta + \delta)}{\sin(\alpha + \beta + \delta + \varphi)} \cdot \cos(\varphi)$$
(33)

The normal force on the blade is now:

$$N_2 = \frac{C \cdot \cos(\varphi)}{\sin(\alpha + \beta + \delta + \varphi)} \cdot \cos(\delta)$$
(34)

Equations 33 and 34 show that the normal force on the shear plane tends to be negative, unless the sum of the angles $\alpha+\beta+\delta$ is greater than 90°. Since the shear angle β and the external friction angle δ are small, the probability of getting a negative normal force here is high. Until now only the total normal force has been taken into consideration, but of course this normal force is the result of integration of the normal stresses on the shear plane. One could consider that cutting is partly bending the material and it is known that with bending a bar, at the inside (the smallest bending radius) compressive stresses will be developed, while at the outside (the biggest bending radius), tensile stresses are developed. So if the normal force N₁ equals zero, this must mean that near the edge of the blade tensile stresses (negative) stresses develop, while at the outside compressive (positive) stresses develop. So even when the normal force would be slightly positive, still tensile stresses develop in front of the edge of the blade. The normal force on the blade however is always positive, meaning that the curling type of cutting process will never occur in rock. In rock both the flow type and the tear type of cutting mechanism may occur.

THE MECHANISMS IN CLAY AND ROCK

In both clay and rock cutting the flow and tear type mechanisms may occur, in clay also the curling type of cutting mechanism. Clay cutting is dominated by adhesion and cohesion (external and internal shear strength), while rock cutting is dominated by internal shear strength and internal and external friction. There is however one soil mechanical parameter that has not yet been discussed, but it has a great influence on both clay and rock cutting. This

parameter is the tensile strength. As discussed before, the normal force on the shear plane might become negative and even if it is slightly positive tensile stresses can develop in front of the edge of the blade. If a material has no tensile strength it cannot handle tensile stresses and it will break, so tensile failure will occur.



Figure 5. The condition under which tensile failure occurs.

Figure 5 shows the Mohr circle of the conditions causing tensile rupture. It can be seen that the rupture occurs on a surface 45° downwards with respect to the shearing surface as is shown in figure 5. Tensile rupture will occur if:

$$\sigma_n - \tau_c \le \sigma_t \tag{33}$$

If the center of the Mohr circle, the so called hydrostatic pressure, is considered to be the normal stress in a point at the shear plane, then deducting the radius of the Mohr circle, which is the cohesion or shear strength, will give the point on the left where the Mohr circle crosses the horizontal axis. If this point is left of the tensile strength in Figure 5, tensile failure will occur, if its on the right, no tensile failure but shear failure will occur. The ratio between the shear strength and the tensile strength is very important. If the tensile strength is much bigger than the shear strength, usually the cutting process will be ductile (flow type), if the tensile strength is much smaller usually the cutting process will be brittle.

If the average stresses on the shear plane are considered, this can be rewritten as an equation of forces:

$$N_1 - C \le T \tag{34}$$

If clay is considered, with an angle of internal friction and a clay/steel friction angle of zero, the following condition can be derived with respect to tensile rupture:

$$\frac{A - C \cdot \cos(\alpha + \beta)}{\sin(\alpha + \beta)} - C \le T$$
(35)



Figure 6. Tensile rupture as described by Hatamura and Chijiiwa (1975-1977)

Figure 6 shows a typical tensile rupture as noticed by Hatamura and Chijiiwa (1975-1977). First a tensile rupture will occur on plane 45 degrees downwards from the shear plane. Secondly a rupture to the free surface perpendicular with the first rupture will occur. From this condition it can be seen that the cohesive force on the shear plane is limited. The maximum cohesive force can be calculated by replacing the less then sign by the equal to sign.

On the blade also a critical condition may occur. The mechanism depends on the ratio between tensile strength and shear strength. If the tensile strength is greater then the shear strength, the normal force on the blade N_2 can become negative. If the tensile strength is smaller then the shear



Figure 7. The condition under which curling of the chip occurs.

strength a tensile rupture may occur under an angle upwards with respect to the blade. A more probable situation is a curling chip. Figure 15 shows the critical conditions for this case. The adhesive force is limited by this condition and can be calculated by replacing the less then sign by the equal to sign. This way also the size of the contact area between the clay and the blade can be calculated. The critical condition is:

$$\frac{C - A \cdot \cos(\alpha + \beta)}{\sin(\alpha + \beta)} - A \le 0$$
(36)

To interpret the conditions derived, three cases can be distinguished:

- 1. The condition as mentioned in equation (35) is satisfied. This may occur when cutting a very thick layer of clay. The failure mechanism will be of the "tear type".
- 2. Neither the conditions of equation (35) nor of equation (36) are not satisfied. This occurs when cutting a medium thick layer of clay. The failure mechanism will be of the "flow type".
- 3. The condition mentioned in equation (36) is satisfied. This may occur when cutting a very thin layer of clay. The failure mechanism will be of the "curling type".



Figure 8. The failure mechanisms.

In the case of rock the conditions are similar as with clay, only rock has no adhesion but it does have internal and external friction.

If rock is considered, the following condition can be derived with respect to tensile rupture:

$$\frac{-C \cdot \cos(\alpha + \beta + \delta)}{\sin(\alpha + \beta + \delta + \varphi)} \cdot \cos(\varphi) - C \le T$$
(37)

If rock is considered under high hydrostatic pressure, the following condition can be derived with respect to tensile rupture:

$$\frac{-C \cdot \cos(\alpha + \beta + \delta)}{\sin(\alpha + \beta + \delta + \varphi)} \cdot \cos(\varphi) - C + F_{hydr} \le T$$
(38)

When a tensile failure occurs, water has to flow into the crack, but the formation of the crack goes so fast that cavitation will occur. Now under atmospheric conditions, the compressive strength of the rock will be much bigger than the atmospheric pressure, usually the rock will have a compressive strength of 1 MPa or more while the atmospheric pressure is just 100 kPa. Strong rock may have compressive strengths of 100s of MPa's, so the atmospheric pressure and thus the effect of cavitation in the crack can be neglected. However in oil drilling at water depths of 3000 m nowadays plus a few 1000's m into the seafloor, the hydrostatic pressure could increase to values higher than 10 MPa causing softer rock to behave ductile where it would behave brittle under low hydrostatic pressures.



Figure 9. Specific energy for different rocks.



Figure 10: Specific energy for different clays.

DISCUSSION AND CONCLUSIONS

In explaining the failure mechanisms when cutting clay and rock it is assumed that the stresses are constant over the shear plane and over the blade. It has been observed by Hatamura and Chijiiwa (1975-1977) that this might not be true in all cases. The method described in this paper however permits the study of the phenomena occurring during the failure of clay and rock in a simple way.

The three possible failure mechanisms can be distinguished and the most important parameters on which the occurrence of a mechanism depends are described. Figure 11 shows these mechanisms as a function of the layer thickness.



Figure 11. The horizontal cutting force as a function of layer thickness.

Figure 9 shows the specific energy for clay and rock, based on the theory of this paper. For clay (a compressive strength upto about 400 kPa) a layer thickness of 10 cm is taken. For the rock different ratios between the compressive strength and the tensile strength have been taken. It is assumed both for clay and for rock that the cohesion or shear strength equals half the compressive strength. This means that an R=2, ratio compressive strength/tensile strength, means that the tensile strength is equal to the shear strength. In the case of R=16, the tensile strength is 1/8 of the shear strength. Both Figure 9 and 10 are for cutting under atmospheric pressure. It will be clear from figure 9 that ductile failure causes a much higher specific energy than brittle failure. In the graph this is about a factor 4-5 going from R=16 to R=2. A high R means that the rock has many fractures before cutting. Cutting at great water depths has the effect of increasing the tensile strength of the rock, resulting in a much higher specific energy and apparently ductile cutting of rock. Figure 10 shows the effect of the layer thickness on the specific energy when cutting clay. A thick layer increases the probability of having the tear type of cutting process, reducing the specific energy.

In both figures 9 and 10 the cohesion is considered to be the compressive strength divided by 2. For a compressive strength ranging from 10-200 kPa the adhesion is considered to be equal to the cohesion and $\varphi=0$ and $\delta=0$. For a compressive strength ranging from 200-400 kPa the adhesion is constant at 100 kPa and $\varphi=0$ and $\delta=0$. For a compressive strength ranging from 400-3300 kPa the adhesion is considered to decrease according to $\tau_a=113.8$ - $0.0345*\sigma_c$ and $\varphi = (\sigma_c-400)/2900*15$ and $\delta=0.67*\varphi$. For a compressive strength above 3300 kPa the adhesion is considered to be 0 kPa and and $\varphi=15^\circ$ and $\delta=10^\circ$. Of course these are assumptions, but they match the experience of the industry.

The general model as shown in this paper is not only suitable for the cutting of sand, but it can also be used for the cutting of clay and rock. Currently the rock cutting processes are being simulated with DEM (Discrete Element Modeling) software, which will be compared with the model from this paper. The result will be published soon.

LIST OF SYMBOLS USED

А	Adhesive force on the blade	Ν
С	Cohesive force on shear plane	Ν
F	Cutting force	Ν
G	Gravitational force	Ν
K_1	Grain force on the shear plane	Ν
K ₂	Grain force on the blade	Ν
Ι	Inertial force on the shear plane	Ν
N_1	Normal grain force on shear plane	Ν
N_2	Normal grain force on blade	Ν
S_1	Shear force due to internal friction on the shear surface	Ν
S_2	Shear force due to soil/steel friction on the blade	Ν
Т	Tensile force	Ν
v	Cutting velocity	m/s
W_1	Force resulting from pore under pressure on the shear plane	Ν
W_2	Force resulting from pore under pressure on the blade	Ν
α	Blade angle	rad
β	Angle of the shear plane with the direction of cutting velocity	rad
τ	Shear stress	N/m ²
τ_{a}	Adhesive shear strength (strain rate dependent)	N/m ²
τ_{c}	Cohesive shear strength (strain rate dependent)	N/m ²
$\sigma_{\rm n}$	Normal stress	N/m ²
σ_{t}	Tensile strength	N/m ²
φ	Angle of internal friction	rad
δ	Soil/steel friction angle	rad

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